Re-representation Processes in Analogical Reasoning

Bachelor’s Thesis

by

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Abstract

Analogy-making is a highly creative and flexible process. It is assumed that human reasoners are able to re-represent initial descriptions of items in order to achieve a coherent mapping. Empirical evidence for this theory, however, is sparse. Furthermore, most models of analogy-making do not incorporate redescription processes. This thesis discusses the need for re-representation and analyses a selection of models with regard to their flexibility. The domain of geometric proportional analogies is proposed for studying redescription processes and an empirical study is devised, which supports the assumption that human reasoners are capable of re-representation in the context of proportional analogies.
Introduction

Analogy-making is a basic feature of human cognition. The ability to understand that “this is like that” (Ross, 1987) plays an important role in our everyday lives. We use analogy to solve problems and explain new domains by transferring knowledge from a similar problem or domain we are familiar with (base) to the yet unfamiliar target. This process is highly creative and flexible.

Standard models of analogy making, however, often fail to represent this flexibility. The initial representations for source and target analog, for example, are in most cases hand coded to fit the system’s needs and cannot be changed in the course of the analogy-making. But re-representation of an analog can be crucial for finding the right mapping in real world problems. Although this seems quite intuitive, empirical data about the re-representation abilities of human problem solvers is sparse. This might be traceable to the difficulty of assessing mental representations and the problem of finding ambiguous material, which is nevertheless structurally clearly definable.

After presenting an overview of the basic principles of analogy in section 1 there will be a short explanation of the importance of re-representation processes in section 2. Then an examination of current analogy models in section 3 is followed in section 4 by details about the domain of geometric analogy problems, which are used in section 5 for the empirical study on human re-representation abilities. A final discussion can be found in section 6.
1 Basic principles of analogy

An analogy is a comparison between two things, which are presumed to be similar in some respect. Valuable analogies focus on the items’ structures rather than on superficial similarities. So, a good analogy between a cloud and a sponge would highlight the fact that both can been seen as a water reservoir, not that they are round and fluffy (Gentner, 1988). However, the concept of analogy is fuzzy. From a less restricted point of view one could also include general similarity comparisons or the use of schemata (Gick and Holyoak, 1983) and metaphors into the category of analogical processes.

Usually, by using analogies, we look at a new concept, situation or domain (henceforth called the target) in terms of one we are familiar with (the base or source). The transfer of knowledge from source to target provides a means of coping with the thus far unfamiliar topic.

1.1 Types of analogy

Concentrating on the strict, structural analogies, there are three types to distinguish (Indurkhya, 1989):

1.1.1 Proportional analogy

Proportional analogies are of the clearly defined form \( A \text{ is to } B \text{ as } C \text{ is to } D \) (or short \( A : B :: C : D \)). They can be used in several domains ranging from word problems (dog : puppy :: cat : kitten) to geometric figures (Evans, 1968; O’Hara, 1992). This sort of task is often used in intelligence tests, where in the context of A, B and C an appropriate choice for D has to be selected.

1.1.2 Predictive analogy

Predictive analogies help us to understand something about a new domain. When a structurally similar source is found, it can be used to draw inferences about the new domain (Gentner, 1983). This process is also known as analogical reasoning. A famous example is the analogy “The atom is like the solar system” (Rutherford).
1.1.3 Analogical problem solving

Analogical problem solving is a strategy commonly used in our daily lives. It is based on the use of examples. Remembering the solution to an old problem helps solving a new one, generally in the same domain (Vosniadou and Ortony, 1989). For example, mathematical problem solving (Novick and Holyoak, 1991) or the development of computer code (Anderson and Thompson, 1989) are often accomplished with continued reference to known solutions of similar problems. In AI this approach is termed Case-based reasoning (e.g. Kolodner, 1993):

1.2 Subprocesses of analogy

The process of analogy-making can be divided into several subprocesses. The most extensive listing of these would include:

1. **Encoding** of perceptual information to construct representations of the analogs (or in some cases only the target)
2. **Retrieval** of a useful source from LTM. Typically, memory access is driven by superficial similarities (Holyoak and Koh, 1987).
3. **Mapping** of corresponding source and target elements. The mapping process is typically guided by structural similarity (Gentner, 1989).
4. **Transfer** of knowledge from source to target
5. **Evaluation** of the constructed mapping and the inferences drawn
6. **Learning** an abstract concept or schema from both analogs that can be applied to future problems (Hummel and Holyoak, 1997)
7. **Re-representation** of an analog. A change of the item’s description might render it more useful for the analogy.

Models of analogy-making are subject to a number of design choices. Up to now, no model has provided a complete picture of the issue, instead most focus on the mapping process (including some means of transfer and evaluation). There have been attempts to model retrieval as well (Thagard, Holyoak, Nelson and Gochfeld, 1990; Forbus, Gentner and Law, 1995) and some systems exhibit a rudimentary learning ability (Hummel and Holyoak, 1997). Nevertheless, a unified account of all subprocesses is not yet in sight.

The role of the re-representation process in analogy-making is most controversial. Some researchers do not consider it at all. Others differ about
where to put it. Does re-representation only come into play when the evaluation reveals uselessness of the preceding mapping (Yan, Forbus and Gentner, 2003) or is it a process interacting continuously with the representation-building and mapping (Chalmers, French and Hofstadter, 1992). The question of the degree of interaction is applicable to all subprocesses. Some theorists presume a wholly modular organization, with the individual subprocesses being independent of each other and running in a serial fashion (Gentner, 1989) Recent publications, however, indicate that human flexibility relies on interacting processes.

2 The need for re-representation

Analogy-making clearly is a highly creative and flexible process. In order to perceive two situations as analogous the reasoner has to highlight certain aspects of the two analogs that can be related to each other. These are not necessarily the most salient features. For example, if you were advised to think about a box of chocolates you might envision the heart-shaped present you got on Valentine’s Day. On the other hand, when someone told you that “Life is like a box of chocolates” you would probably come up with the idea of “never knowing what you are going to get”. Obviously, the representation of a particular item is influenced by the context in which it appears (Chalmers et al., 1992; Dastani and Indurkhya, 2001). The reasoner’s current goals also play an important role in the representation process. To illustrate the properties of light, it is convenient to compare the phenomenon to waves in one context but to particles in another.

Especially suitable for studying the context effects on representation-building is the domain of proportional analogy problems. Have a look at Figure 1 (page 7). The Star of David appears in three different geometric analogy problems. It becomes apparent that there are several possible descriptions for this figure. Depending on the context, provided by the other items in the analogy, the star is decomposed into two triangles, a hexagon with six small triangles or three
rhomboids. The flexibility human reasoners exhibit in the use of these representations will be investigated in the empirical part of this thesis.

Figure 1: Three proportional analogies containing the Star of David figure. In order to solve each problem, three different representations of the ambiguous figure have to be assumed. (Adopted with modification from Indurkhya (1989))

3 Analogy models

In the past decades a variety of analogy-making models has been published. Although these models differ in many aspects ranging from the analogy subprocesses investigated (retrieval, mapping, transfer, learning) to the implementation techniques used (symbolic, connectionist or hybrid approach), most of them do not incorporate any mechanism for re-representation. Initial representations of source and target analog are rigid and cannot be changed by the system.

I will now give a short overview of some models and especially their re-representation capabilities. Of course, an exhaustive examination of the vast field is not possible here, but I try to present a representative selection.
3.1 Classical models without re-representation

3.1.1 SME

Gentner’s Structure Mapping Engine, SME (Falkenhainer, Forbus and Gentner, 1986), is probably the best-known example of an analogy-making model. It is based on the Structure Mapping Theory SMT (Gentner, 1983), which today still constitutes the base of many classical analogy models. Several important assumptions underlie Gentner’s model. First, the sub-processes involved in analogy-making are separable from one another. SME focuses on mapping in isolation from other processes like representation-building and retrieval. Second, SME relies solely on structural similarity. Predicate calculus descriptions of source and target are taken as input and a set of local matching hypotheses for elements of the two analogs are created. These candidate matches are then evaluated and integrated into a coherent mapping. The mapping process is guided by two structural constraints.

1. relational matches are favored over attribute matches
2. matches of higher-order relations are preferred, supposing that systems of mutually interconnected relationships are of more centrality to the problem than isolated predicates. (systematicity principle)

When a satisfying mapping has been achieved, additional information from the base can be transferred to the target domain.

Early versions of SME allowed only equal predicates to be matched. So, the system’s performance depended crucially on the input representations. Since then, the model has been gradually refined. In 2003, Yan, Forbus and Gentner even introduced a “theory of rerepresentation in analogical matching”. Some mechanisms were established that were supposed to make SME more flexible by rewriting its predicate calculus formulas when necessary. For this purpose a re-representation module is inserted into the system. This is certainly a positive development, but upon closer consideration the refinements only attempt to repair the model’s intrinsic flaws. SME is far from demonstrating human flexibility.
3.1.2 ACME

The ACME (Analogical Constraint Mapping Engine) program devised by Holyoak and Thagard (1989) models analogical mapping by constraint satisfaction. As the first model to apply a connectionist technique ACME takes input representations of source and target and builds a network with the nodes corresponding to all possible matches of source and target elements and the links between nodes implementing the constraints to be satisfied. Competing hypotheses are linked by inhibitory connections, while consistent hypotheses support each other with excitatory links. Unlike SME, the system relies not purely on structural information but considers also semantic and pragmatic constraints. A spreading-activation relaxation algorithm is run on the network. When the network settles to a final state, active nodes represent winning matches.

Unfortunately, ACME shows no re-representation capabilities what so ever. Not only are the predicate calculus representations of source and target analog hand-tailored to the problem, even the degree of semantic similarity and the practical relevance of elements (implemented via connections from a semantic and a pragmatic node respectively) are predefined by the experimenter. Interestingly, Holyoak and Thagard (1989, page 351) acknowledge that “it will often be necessary to interweave the mapping component with strategic manipulation of the representations of the source and target.” But they give no hint of how this mechanism could be incorporated into their model.

3.1.3 LISA

LISA (Learning and Inference with Schemas and Analogies) is an integrated theory of analogical access and mapping (Hummel and Holyoak, 1997). The model’s unique architecture combines the strengths of symbolic and connectionist approaches, making the system both sensitive to structure and flexible.

The architecture can be roughly divided into a long-term memory (LTM) and a working memory (WM) component. Every proposition is encoded in LTM by a hierarchy of structure units of three types: propositional units (top level), sub-
propositional units (intermediate level) and predicate and object units (bottom level). These structure units do not carry meaning in themselves, but the connections of predicate and object nodes to corresponding semantic units in the WM layer give rise to a representation of predicates (roles) and objects (role fillers) as distributed patterns of activation in WM. The more similar two entities are the more semantic nodes they will share.

Structure sensitivity is achieved through dynamic binding. Rather than allocating a new node for each possible grouping of an object and a predicate, the binding of fillers to their roles is represented in WM by synchronous oscillation. A set of mapping connections between structure units of the same type completes the architecture.

Retrieval of a source analog is achieved through spreading activation. The activation of a target analog generates specific patterns of activation on the WM semantic units. This in turn activates other propositions since the semantic units are shared by all analogs. Those LTM analogs that become most activate apparently share some features with the target and are considered as possible source analogs. Mapping is closely related to the retrieval process. Units that are active simultaneously receive a strengthening of their mapping connections. This way, corresponding structure units are found, the right mapping is “learned”.

Although this model exhibits some flexibility in its dynamic computation of semantic similarity, it is nevertheless incapable of re-representation. The propositions given as input are rigid, and also the semantic features associated with each object and predicate are fixed in advance. LISA is insensitive to context and will always produce the same outcome for a given problem.

3.2 Models with re-representation abilities

3.2.1 Copycat

The Copycat project belongs to a family of models from the Fluid Analogies Research Group (Hofstadter and Mitchell, 1994). The most important characteristic of these models is the integration of representation-building and
mapping processes. Hofstadter and his colleagues are convinced that high-level perception - the extraction of meaning from raw sensory data in form of concepts - is inseparable from higher cognitive processes.

In a microdomain of letter-strings, Copycat gradually builds its own representations. Bottom-up and top-down processes interact to create a coherent analogy.

Copycat’s architecture consists of three major components:

1. the *Slipnet*, a semantic network corresponding roughly to LTM, which contains all concepts known to the system (in the case of the letter domain these are concepts like identity or successorship of letters)
2. the *Workspace*, corresponding to WM
3. the *Coderack*, a pool of agents (called codelets) waiting to be called to action

In order to solve a proportional analogy problem like $abc : abd :: ijjkkl : ?$ a number of codelets works in parallel. Each codelet locally performs a particular type of task, e.g. determine relations between neighboring letters, chunk letters into groups or build correspondences between source and target elements. Although codelets are selected nondeterministically, the probability of a special type to run successfully is influenced by top-down processes. While processing of the input strings proceeds, concepts become activated in the *Slipnet*. This emphasis on certain concepts in turn guides the codelets’ further activity. In this way, influences from context (i.e. the other strings) are taken into account.

A mechanism called *computational temperature*, which constantly evaluates the quality of the current representation, is used to control the degree of random exploration. In the beginning the *temperature* is high resulting in an exploration of many different paths. The more structure is discovered by the codelets, the lower the *temperature* becomes until the system finally converges on one particular representation of the situation. Existing representations can even be destroyed with *breaker codelets*. Copycat convincingly integrates the representation-building and mapping processes and is capable of re-representation.
3.2.2 AMBR

AMBR (Associative Memory-Based Reasoning) (Kokinov, 1994a; Kokinov and Petrschemaov, 2001) is a complex model of analogical problem solving based on a cognitive architecture called DUAL (Kokinov, 1994b). The authors’ ambition is not only an integrative account of the analogy subprocesses but also the investigation of analogy-making in a broader cognitive context. A DUAL-system consists of a large quantity of micro-agents, whose interaction provides a means for dynamic emergent representation and computation. The simple agents are dualistic in nature: their symbolic part represents a piece of knowledge, while the connectionist part (the agent’s activation level) stands for the relevance of this knowledge to the current context. In the system’s LTM component, loosely grouped coalitions of agents allow for distributed representations of objects, concepts and episodes. All agents, whose activation level exceeds the threshold, enter WM. Furthermore, the agent’s activation level determines its individual processing speed, resulting in faster processing of relevant information.

AMBR integrates retrieval, mapping and transfer. These processes run in parallel and can influence each other. Knowledge is gradually retrieved from memory via a spreading activation mechanism. The sources of activation are those agents that represent the environmental elements perceived and the current goals. Since the decay rate is very low, residual activation in the network can produce priming effects. As soon as the first potential source elements enter WM, the mapping process starts. It is based on local mechanisms such as marker passing and construction of temporary matching-hypothesis-agents. Partial mappings influence the further retrieval process.

Whereas early versions of AMBR used rigid representations of episodes, the newer model AMBR2 is claimed to be capable of re-representation (Kokinov, Petrov, 2000). In fact, the system demonstrates an instantiation mechanism: Driven by some unmappable target elements, the description of a base episode can be extended with information inferred from generic knowledge. This way, the system certainly gains flexibility, but it is still not capable of producing a complete structural redescription. The early influence of mapping on the
retrieval process seems a better candidate, though. If the “right” representation is stored in LTM it might eventually be retrieved, even if it was not the first choice. However, I would expect intense priming on a particular representation, as it is used in the experiment described in this thesis, to strongly inhibit the retrieval of a new description.

3.2.3 PAN

In 1992 O’Hara proposed an AI model called PAN that solved proportional analogies in the domain of geometric figures (A is to B as C is to D). The focus of this project was indeed the incorporation of redescription processes. Initial descriptions given to PAN were able to change and, in interaction with one another, create a coherent picture of the proportional analogy.

Using an algebraic representation PAN distinguishes between a sensorymotor level and a concept level. From a single sensorymotor network representing one item in the analogy several conceptual networks can be derived accounting for the different possibilities to represent this figure. Each conceptual network constitutes a subalgebra of the sensorymotor algebra and contains only a subset of the original objects and operators.

As input PAN takes a fixed description of item A, represented in a tree-structure with operators on the non-terminal nodes and objects on the leaf-nodes, as well as a sensorymotor network description for item B and C each. A successful solution for the proportional analogy problem involves several steps:

1. **transformation:** create a description for B which is a transformation of A
   
   \[ B \rightarrow t(A) \]

2. **mapping:** create a description for C via an isomorphic mapping from A to C
   
   \[ C \rightarrow m(A) \]

3. **apply mapping:** extend mapping m to m’ upholding the isomorphism condition by adjusting it to the special objects and operators in B. Get a description of D by applying m’ to the description of B.
   
   \[ D \rightarrow m'(B) \]
The program’s capability of backtracking is the basis for its re-representation potential. So, when a dead end is reached, the system can undo its last steps and produce an alternative solution.

4 The domain of geometric analogy problems

A domain for studying re-representation processes has to be clearly definable. But, on the other hand, it must be demanding enough to reveal creativity and flexibility in problem solving. The domain of geometric proportional analogy problems is ideal for this purpose. Finding a solution to a geometric analogy problem becomes possible only through the integration of representational and reasoning processes. General world knowledge, however, is excluded and so these problems are feasible also for computer models. Formally, proportional analogies can be characterized as isomorphisms (Cornuejols, 1997; Klix, 1993; as reviewed in Schmid, Burghardt and Wagner, 2003). The descriptions of the individual items are related via mappings \((A \rightarrow C; B \rightarrow D)\) and transformations \((A \rightarrow B; C \rightarrow D)\).

A simple algebra can be used for describing geometric figures. Formally, an algebra is a, possibly infinite, set of objects together with a set of operators defined over those objects. Simple objects, e.g. triangles, are combinations of line segments. Complex objects can be built from simple objects using the operators.

Only five different operators are needed to describe the items used in the experiment reported in section 5 of this thesis. These are (adopted with changes from O’Hara, 1992):

- **ROTATE\([d](\text{object})\)**: Rotates the object \(d\) degrees around its center
- **SLIDE\([x,y](\text{object})\)**: Moves the object \(x\) steps horizontally and \(y\) steps vertically
- **COPYROT\([m, a, d](\text{object})\)**: Rotates \(m\) copies of the object incrementally with \(d\) degrees around axis \(a\)
- **MIRROR\([a](\text{object})\)**: Mirrors the object at axis \(a\)
- **GLUE(\text{object1, object2})**: Combines two objects to a complex figure
Each description for an analogy item can be modeled as a tree with the internal nodes labeled with operators and the leaf nodes with simple objects. A bottom-up reading of the tree yields the complex object. Simple objects are assumed to exist originally in a centered position (<0,0>) on the screen. An example for two descriptions of the Star of David figure can be found in Figure 2.

![Figure 2: Two descriptions for the Star of David figure](image)

5 Experiment

This experiment was designed to demonstrate re-representation processes in solving proportional analogies. An initial description of a geometric figure, which is imposed upon the subject using a visual prime, has to be changed to arrive at a correct solution of a proportional analogy problem.

5.1 Method

5.1.1 Material

The experiment was implemented in HTML and JavaScript. A printout of the individual screens can be found in the appendix.
The materials used can be roughly divided into three categories: proportional analogy problems, selection tasks and animations. All of these were based on simple geometric forms (black line drawings) as described in section 4. There was only one task per screen.

**Proportional analogy problem:** In the upper half of the screen a proportional analogy problem is shown. This consists of three geometric figures (position A, B and C) and a question mark in the D position. Below are four items (in size and makeup equal to the analogy items) to choose a solution from.

**Selection task:** A single geometric figure is presented in the upper half of the screen (centered). Below are four items to choose from, just like on the proportional analogy screen. The instruction says to choose the item that best relates to the one above.

**Animation:** A single geometric form is presented in the center of the screen. This one is bigger than the items in the analogy or selection context. Beneath the figure are buttons for starting the animation and resetting the item to its original form. When the animation is set off by clicking the corresponding button, the object on the screen turns out to be composed of several parts, which are then moving into different directions. The first animation shows the Star of David that is then revealed to consist of two triangles. The second animation shows a shape dividing into four trapezoids.

**5.1.2 Design**

The experiment was conducted with four conditions. Apart from the *control condition*, which was used to find the preferred solutions for the proportional analogy problems without priming, there was one *test condition* including all steps of representation, confirmation, representation retrieval and re-representation (within-subject design) and two supplementary conditions splitting the test condition into a representation retrieval and a re-representation part (between-subject design).

The participant’s choice of a solution (dependent variable) was investigated in two different contexts (independent variable).
5.1.3 Procedure

The experiment was conducted over the internet. Subjects having access to a computer with internet connection could log in anytime. Since the experiment was implemented in standard HTML and JavaScript it was compatible with most web-browsers (tested on Mozilla 1.6, Internet Explorer 5 and Opera 7). There was no time limit for solving the individual problems.

Participants were assigned to one of the four conditions by the system. The distribution was accomplished in a stochastic manner, because more participants were desired for the test condition than for the control condition and the split conditions (40% and 20% respectively).

The main idea of demonstrating re-representation processes involved 4 steps:

1. **priming**: a particular representation of the ambiguous figure was imposed on the subject with an animation
2. **consolidation**: the impression of the primed representation was strengthened with an analogy problem with items structurally similar to the crucial figure
3. **representation test**: the current representation of the crucial figure was requested from the participant
4. **re-representation**: the context of an proportional analogy problem forced the participant to re-represent the crucial figure

All groups first received an example. This was a simple proportional analogy problem with geometric figures (adopted with changes from…). When subjects had picked one of the possible solutions and proceeded to the next screen they were provided with the correct solution and an explanation of the reasoning process.

After this warm-up the real experiment started. Now, the test group was shown a screen with an ambiguous geometric figure (the Star of David) and the advice to watch the animation, because it might help them to solve the following problems. On clicking a button, the animation was set off and the star divided into two triangles that moved apart. This method should prime the subjects on a particular description of the figure. This representation was then reinforced on the subsequent screen by presenting a proportional analogy problem with different shapes but a similar underlying structure: a figure made up of two
identical simple forms. Before getting to the critical re-representation stage, a
test for the subjects’ current representation of the ambiguous item (the Star of
David) was inserted. Participants had to choose from four items the one they
would relate to the Star of David. These four items included the two-triangle-
representation, a conceptually similar figure (a star made up of three lines), a
superficially similar figure (a hexagon surrounded by six squares) and the
solution for the following re-representation problem (a hexagon). The final part
of the first block of problems was a proportional analogy task with the
ambiguous Star of David figure in the C position. But in this case, the context
provided by A and B called for an alternative representation of the star figure: a
hexagon surrounded by six little triangles. The possible solutions to choose
from were the same as in the preceding representation test, but in a different
order. Only those participants, that were able to change the representation of
the Star of David imposed on them before into a new one that fitted the current
context, could succeed in this analogy problem. The whole process of priming,
consolidation, representation test and re-representation was then repeated with
a second set of symbols. A complete run of the experiment’s test condition is
shown in the appendix.

Split condition 1 and 2 were devised to rule out undesirable interaction effects
between the representation test and the re-representation stage. Split condition
1 was used to test for the participant’s representation of the crucial item
immediately prior to encountering the re-representation problem. This
condition involved all stages of the test condition except for the final re-
representation problem. Split condition 2 left out the representation test
instead.

Participants in the control condition did not see any animations. After the
example, this group received the four proportional analogy problems, used in
the other conditions, on four successive screens.
5.1.4 Participants

The experiment was promoted in a selection of internet gaming communities. A call for participation was also sent via the mailing list of the cognitive science program in Osnabrück.

A total of 150 subjects participated in the experiment, three of which had to be excluded since they did not submit solutions to all problems. From the remaining 147 participants 107 (72.8%) were male and 40 (27.2%) were female. The average age was 23 years, in a range from 13 to 54 (but 90% of the subjects were aged between 16 and 35). 58.6% of the participants indicated that they were familiar with the sort of problems they had to solve in the experiment. The reported self-rating of expertise displayed a mean of 3.27 on a scale from 1 (novice) to 5 (expert).

The final distribution of participants to conditions was as follows: 64 in the test condition, 31 in split condition 1, 27 in split condition 2 and 25 in the control group.

5.2 Results

From the 150 entries collected in the database, three had to be excluded from analysis, because they contained missing values.

Evaluated were then the results from the representation test and the re-representation stage. Table 1 gives an overview of the results obtained. The participants’ preferences for particular representations of the crucial item are compared in two situations. In the prime context, assessed through selection of the best-matching item at the representation test, the majority of participants (93.55% in split condition 1 and 93.75% in the test condition, averaged over both blocks) chose the primed representation of the ambiguous item.

Whereas in the analogy context, assessed through selection of the best-matching item at the re-representation stage, most participants abandoned the primed representation and chose the correct analogy solution instead (92.59% in split condition 2 and 89.84% in the test condition, averaged over both
blocks). Figure 3 shows the preferred choices in the prime and analogy context for the test condition.

The difference between the participants’ representations of the crucial items in the prime and the analogy context are highly significant (according to Fisher's exact test the two-tailed p value is less than 0.0001).

<table>
<thead>
<tr>
<th>block</th>
<th>choice</th>
<th>context (split 1)</th>
<th>Fisher’s exact</th>
<th>context (split 2)</th>
<th>Fisher’s exact</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>p</td>
<td>27 (87.1%)</td>
<td>p&lt;0.0001</td>
<td>3 (11.11%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>0 (24 (88.89%))</td>
<td></td>
<td>24 (96.3%)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>p</td>
<td>31 (100%)</td>
<td>p&lt;0.0001</td>
<td>0 (93.75%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>0 (26 (96.3%))</td>
<td></td>
<td>26 (96.3%)</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>p</td>
<td>58 (93.55%)</td>
<td>p&lt;0.0001</td>
<td>3 (5.56%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>0 (50 (92.59%))</td>
<td></td>
<td>50 (92.59%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Frequencies and percentages of participants’ choices for a particular representation of the crucial item. Both the assessments from the split conditions and the test condition are shown. While in the prime context nearly everyone preferred the primed (p) representation, in the analogy context most participants chose the representation that allowed for a correct solution of the analogy problem (a). The significance of these differences is shown by a Fisher’s exact test rendering p<0.0001 for all blocks and conditions.

In the control group overall performance on the analogy problems was very high. All participants gave correct answers to the problems used as representation problems in the other conditions. Only one participant failed to solve the example and the first analogy problem that was used in the other groups for consolidation.
5.3 Discussion

The data collected in this experiment support the intuitive assumption that human reasoners’ analogy-making is a creative and flexible process. A majority of the participants could solve the final analogy problem successfully, although it required a representation of the crucial C item that was completely different from the one they had just learned. The internal description of the figure had to be altered; the item was re-represented.

This hypothesis is most convincingly supported in the test condition. The same persons that had just produced the primed description for the crucial item in the representation test demonstrated re-representation abilities by solving the following proportional analogy problem correctly. The split conditions cannot make an equally strong claim. In that case, half of the participants were never forced to re-represent. The other half received weaker priming, because they skipped the representation test. This might be the reason for the slightly better performance of the split condition 2 group in comparison to the test group.

Some participants reported that they found the analogy problems rather easy. This is not a problem per se. The experiment was designed to provide evidence
for the existence of re-representation processes in analogical reasoning, irrespective of the task’s complexity. Nevertheless, the question whether more difficult problems inhibit the re-representation process could be worth investigating.

A ubiquitous problem is that of mental representations. Even with all the effort devoted to requesting the current representation in the representation test and using the analogical context to elicit a re-representation, the internal processes cannot be fully revealed. For example, there remains the possibility that participants do not relate the ambiguous item occurring in the analogy problem to the figure they saw before. If the item was perceived as new, no re-representation would be necessary in order to solve the analogy problem. Putting the ambiguous item in the A or B position of the proportional analogy setup might reduce the probability of this disconnection to happen.

6 Final discussion

The necessity of making models of analogical reasoning more flexible becomes increasingly evident. The empirical data provided in this thesis support the claim for an implementation of redescription processes into analogical models, since human reasoners are apparently able to re-represent initial descriptions, when making an analogy. The nature of the underlying processes, however, remains unexplored. This topic exceeds the scope of this thesis and will need further investigation in the future.
References


Acknowledgements

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Appendix: Material

Here is one complete run of the test condition, which includes all problems used in this experiment. The pictures are scaled down. The original size depended on the participants’ screens and resolutions. But the differences arising from this variability are negligible.

Since the experiment was conducted in German, translations can be found underneath the original boxes (only for the first set of problems).

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**Screen 1: Introduction**

Thank you for your interest in my online experiment. For my bachelor's thesis I investigate particular aspects of analogical reasoning. This is no test of your individual intelligence and skill. Nevertheless, I would ask you to work on the problems seriously. The experiment will take about 10 minutes time. With problems and questions turn to experimenter@gmx.net

If you are interested in background information about this investigation, send a note to the address above. I would be pleased so send you information once the data collection has been finished.

---

Vielen Dank für Ihr Interesse an meinem Online-Experiment!

Der geschätzte Zeitaufwand für dieses Experiment beträgt 10 Minuten.

Bei Problemen oder Fragen wenden Sie sich an experimenter@gmx.net

Falls Sie sich für die Hintergründe der Untersuchung interessieren, schicken Sie eine kurze Notiz an obige Adresse. Nach Abschluss der Datenerhebung werde ich Ihnen dann gerne Informationen zustellen.
Sie bekommen im Folgenden Aufgaben der Form

A : B :: C : ?

A ist zu B wie C zu D

Welches der vier Bilder unten müssen Sie für D einsetzen um die Analogie zu vervollständigen?

1 2 3 4

Screen 2: Example

In the following you will receive problems like this one

A : B :: C : ?

A is to B as C is to D

Which of the four pictures underneath do you have to put in to complete the analogy?

1 2 3 4

submit solution
Antwort 3 ist richtig!
Das kleine Objekt wird vom rechten Rand des großen Objekts in dessen Mitte bewegt.

Wenn Sie das Beispiel verstanden haben, können Sie jetzt mit den eigentlichen Testaufgaben fortfahren.

Screen 3: Example solution

3 is correct!
The small object is moved from the right border of the big object to its center.

If you understood the example, you can now proceed to the real test.
Sehen Sie sich diese Animation an. Sie könnte Ihnen bei der Lösung der folgenden Aufgaben behilflich sein.
Wenn Sie die Animation angesehen haben, klicken Sie auf "weiter".

Screen 4: Prime

Have a look at this animation. It might help you solve the following problems.
When you have seen the animation, press the "proceed" button.
Lösen Sie diese Analogieaufgabe

[diagram]

Wählen Sie eine der Lösungsmöglichkeiten:

[diagram with options]

Screen 5: Consolidation of primed operation

Find a solution to this analogy problem!

[diagram]

Choose one of the possible solutions:

[diagram with options]
Screen 6: Test for mental representation

Which one do you feel is right? (no exact translation possible)

Which of the four choices would you relate to the picture above? Choose one!
Lösen Sie diese Analogieaufgabe

\[
\begin{array}{ccc}
\boxplus & : & \Box \\
\hline
\star & : & \mathbf{a} & : & ?
\end{array}
\]

Wählen Sie eine der Lösungsmöglichkeiten:

1 2 3 4

Lösung abschicken

Screen 7: Re-representation test

Find a solution to this analogy problem

\[
\begin{array}{ccc}
\boxplus & : & \Box \\
\hline
\star & : & \mathbf{a} & : & ?
\end{array}
\]

Choose one of the possible solutions:

1 2 3 4

submit solution

End of first set
Lösen Sie diese Analogieaufgabe

Wählen Sie eine der Lösungsmöglichkeiten:

Screen 9: Consolidation of primed operation
Screen 10: Test for mental representation

Welches würden Sie zuordnen?

Welche der vier Auswahlmöglichkeiten ordnen Sie der oberen Figur zu? Wählen Sie eine!

Screen 11: Re-representation test

Lösen Sie diese Analogieaufgabe

S : S :: S : ?

Wählen Sie eine der Lösungsmöglichkeiten:

Screen 11: Re-representation test

End of second set
**Fragebogen**

Das war es schon fast. Zum Schluss benötige ich nur noch ein paar persönliche Angaben.

**Geschlecht:**
- m
- w

**Alter:**

**Schulabschluss:**
- Bitte auswählen...

**Beschäftigung:**
- Bitte auswählen...

**Hatten Sie schon vorher Erfahrung mit Aufgaben dieser Art?**
- Ja
- Nein

Wie würden Sie Ihre Fähigkeiten in Bezug auf solche Aufgaben einschätzen?
- Anfänger
- Profi

Sind während des Experiments technische Probleme aufgetreten?
- Keine technischen Probleme

**Fragebogen abschicken**
Erklärung

Ich erkläre hiermit an Eides Statt,

- dass ich die vorliegende Studienarbeit selbstständig angefertigt,
- keine anderen als die angegebenen Quellen benutzt,
- die wörtlich oder dem Inhalt nach aus fremden Arbeiten entnommenen Stellen, bildlichen Darstellungen und dergleichen als solche genau kenntlich gemacht und
- keine unerlaubte fremde Hilfe in Anspruch genommen habe.

Berlin, 28.06.2004

Julia Jira