SVM Active Learning

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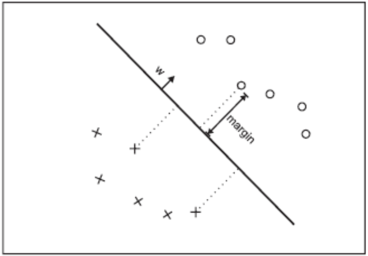
Let us **consider** the following: $\{x_1 ... x_n\}$ are **vectors in** some **space** $X \subseteq \Re^d$. And $\{y_1 ... y_n\}$ are their corresponding **labels**, where $y_i \in \{-1, 1\}$.
SVMs allow one to **project** the original **training data** in space X to a **higher dimensional feature vector** F via a **Mercer kernel** K. Therefore we are considering the set of classifiers of the form:

$$f(x) = \sum_{i=1}^{n} \alpha_i K(x_i, x)$$

Given a kernel K satisfying **Mercer's condition** $K(u, v) = \Phi(u) \cdot \Phi(v)$ where $\Phi : X \rightarrow F$, we can rewrite f as:

$$f(x) = w \cdot \Phi(x), \text{ where } w = \sum_{i=1} \alpha_i \Phi(x_i).$$

A simple linear SVM



One of the commonly used **kernels** is the **radial basis function** kernel:

$$K(u, v) = e^{-\gamma(u-v)\cdot(u-v)}$$

For these kernels one **property** holds: $\| \Phi(x_i) \| = \lambda$ for some fixed λ.

Proof:

$$\| \Phi(x_i) \| = \sqrt{\Phi(x_i) \cdot \Phi(x_i)} = \sqrt{K(x_i, x_i)} = \sqrt{e^{-\gamma(x_i - x_i)\cdot(x_i - x_i)}} = \sqrt{e^0} = 1$$

Let us consider the set of **hypotheses** f, that is the set of **hyperplanes** that **separate** the **data in** the **feature space** F.

$$H = \{f \mid f(x) = \frac{w \cdot \Phi(x)}{\| w \|}, \text{ where } w \in W\}$$

W is the **parameter space** and is **equal** to F
The **version space** V is therefore defined as:

$$V = \{f \in H \mid \forall i \in \{1..n\} : y_i f(x_i) > 0\}$$

Since there is a **bijection** (an exact correspondence) between w and $f \in H$. We redefine V as:

$$V = \{w \in W \mid \| w \| = 1,\ y_i(w \cdot \Phi(x_i)) > 0,\ i = 1..n\}$$

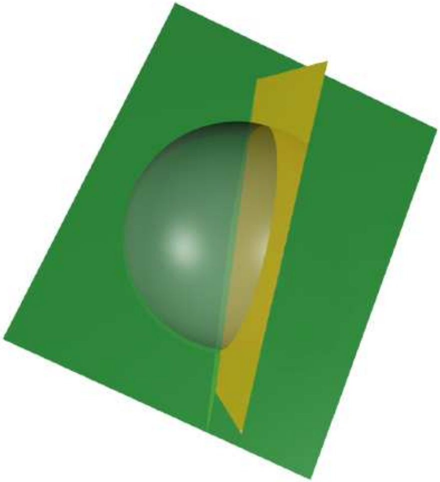
Duality

By definition **points in** W correspond to **hyperplanes in** F.
How about the converse?

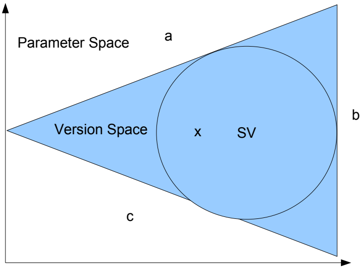
Duality

Suppose we have a new **training instance** x_i with **label** y_i.
Then any **separating hyperplane must satisfy** $y_i(w \cdot \Phi(x_i)) > 0$.
Rather than viewing w as the **normal** of a hyperplane **in** F, we think of $y_i\Phi(x_i)$ as being the **normal** of a hyperplane **in** W
Therefore $y_i(w \cdot \Phi(x_i)) = w \cdot y_i\Phi(x_i) > 0$ defines a **half-space in** W.
$w \cdot y_i\Phi(x_i) = 0$ defines a **hyperplane in** W which acts as one of the **boundaries** to **version space** V.

Version space duality



Version space duality 2D



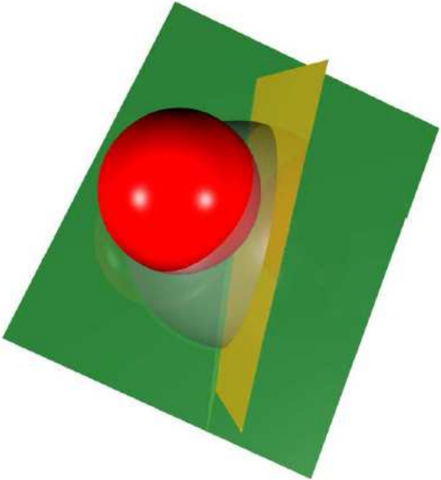
SVMs find the **hyperplane** that **maximizes** the **margin** in F.

$$maximize_{w \in F} min_i \{y_i(w \cdot \Phi(x_i))\}$$
$$\text{subject to: } \parallel w \parallel = 1 \text{ and } y_i(w \cdot \Phi(x_i)) \; > 0 \text{ for } i = 1..n$$

These **conditions restrict** the **solution** to lie **in version space**. **Given** the **duality** between F and W and **since** $\parallel \Phi(x_i) \parallel = 1$, **then** each $y_i\Phi(x_i)$ is a **unit normal vector** of a **hyperplane in** W that **delimits** the **version space**.

We want to **find the point in version space** that **maximizes** the **minimum distance** to any of the **hyperplanes**.
SVMs find the **center** of the **largest radius hypersphere** whose **center** is **in version space** and whose **surface does not intersect** with the **hyperplanes** corresponding to the **labeled instances**.
The **touched hyperplanes** by this **maximal radius hypersphere** correspond to the **support vectors** and the **radius** of the hypersphere is the **margin** of the **SVM**.

An SVM classifier in version space



Let us first define the following:

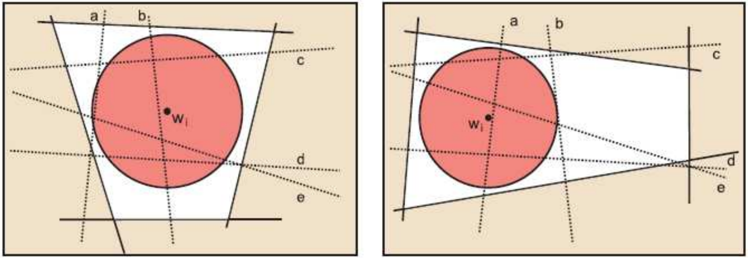
- A pool of **unlabeled instances** U
- An **active learner** l with three components (f, q, X)
- A **classifier** $f : X \to \{-1, 1\}$ trained on **labeled data** X, (and maybe on some of U)
- The **querying function** q(X)

The main difference between an active learner and a passive learner is the querying component q.
How to choose the next unlabeled instance in the query-pool?

`Definition`: $Area(V)$ is the **surface** area that the **version space** V occupies on the **hypersphere** $\| w \| = 1$.
In order to **reduce** the **version space** as fast as possible, we can intuitively choose a **pool-query** that **halves** the **version space**.

A **Simple** method is to **learn** an **SVM** based on the **existing labeled data** X and **choose** as the **next** instance to **query** the pool **instance** that comes **closest** to the **hyperplane** in F.

Simple margin method



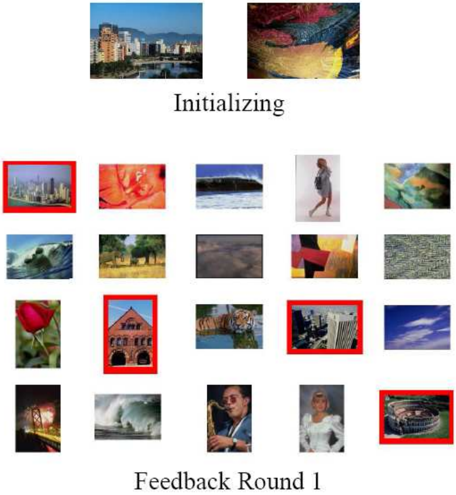
SVM_{Active} algorithm summary

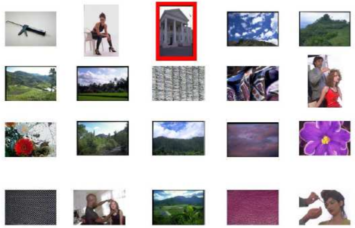
- Learn an SVM on the current labeled data
- If this is the first feedback round, ask the user to label twenty randomly selected images.
- Otherwise, ask the user to label the twenty pool images closest to the SVM boundary.

After the relevance feedback rounds have been performed SVM_{Active} retrieves the top_k most relevant images:

- Learn a final SVM on the labeled data
- The final SVM boundary separates **relevant** images from **irrelevant** ones. Display the k **relevant** images that are farthest from the SVM boundary.

Example





Feedback Round 2

Feedback Round 3


First Screen of Results
Second Screen of Results
Third Screen of Results
Fourth Screen of Results

Fifth Screen of Results

Sixth Screen of Results

Demo

1. Simon Tong, Edward Chang. *Support Vector Machine Active Learning for Image Retrieval*.
2. Simon Tong. *Active Learning: Theory and Applications*. Ph.D. Thesis.
3. URL: `http://ai.stanford.edu/~stong/research.html`