Intelligent Agents

Formal Characteristics of Planning

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Extensions to the slides for chapter 3 of Dana Nau
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Outline

- Semantics of classical planning
- Evaluating planning systems
- Basic Results of Decidability
- Basic Results of Complexity

Semantics of Classical Planning

- Distinction of a syntactical planning problem and what it means
- Analogous to distinction between a logical theory and its models
- Statement of a planning problem: $P = (O, s_0, g)$
- Problem: $\mathcal{P}$ with a state-transition system $\Sigma = (S, A, \gamma)$

Semantics cont.

- Denotational semantics: Interpretation function I such that
  - For every state t of P, $I(t)$ is a state of Σ
  - For every operator instance o of P, $I(o)$ is an action of Σ

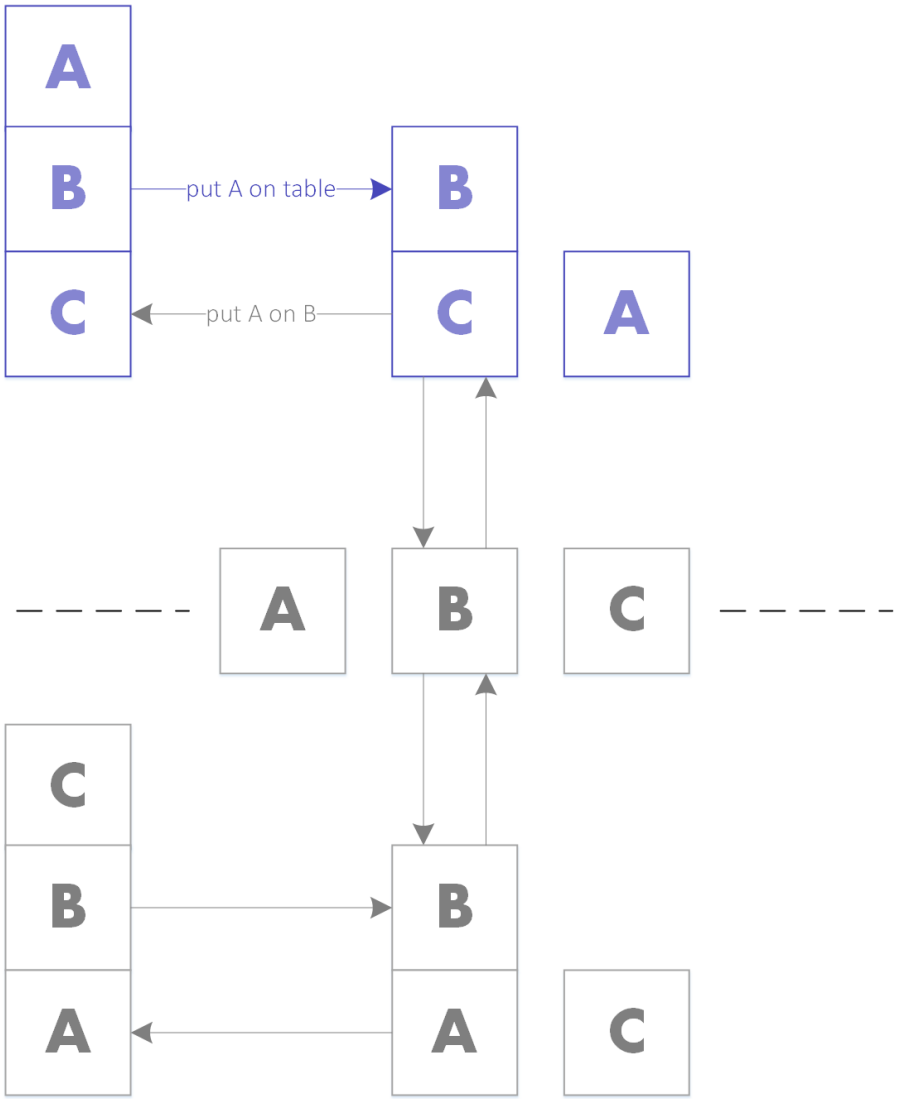
  The pair (Σ, I) is a *model* of P if for every state t of P and for every operator instance o of P holds:

$$\gamma(I(s), I(o)) = I((s - \mathit{effects}^-(o)) \cup \mathit{effects}^+(o))$$

Semantics cont.

- Distinction between syntax and semantics:
  - Syntactic calculus (automated computation)
  - with *soundness* property
- For classical representation the question whether Σ is a model of P is quite trivial; for extended representation it can get complicated (see lectures on deductive planning in FOL)

State-Space Model



- Arc from state s_i to s_j iff s_j can be reached from s_i by performing a single action.

Evaluating Planners

- **Termination** (critical case: no solution exists)
- **Soundness**: every plan returned is a legal sequence of actions to achieve the goal
  implies consistency: each intermediate state appearing in the plan is a legal state of the domain
- **Completeness**: the planner finds a solution, if one exists.
- **Optimality**: the returned plans are optimal (shortest) solutions (typically not considered)
- Expressiveness of the planning language
- Complexity of planning problems in a given representation formalism
- Efficiency of algorithms

Evaluation of Depth-First-Search and Breadth-First-Search

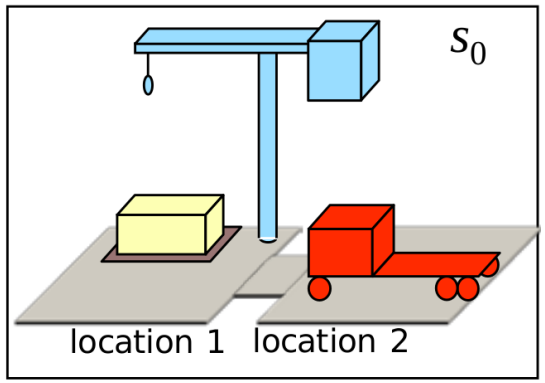
- **Soundness:** A node s is only expanded to such a node s' where (s, s') is an arc in the state space (application of a legal operator whose preconditions are fulfilled in s)
- **Termination:** For finite sets of states guaranteed.
- **Completeness:** If a finite length solution exists.
- **Optimality:** Depth-first no, breadth-first yes
- worst case $O(b^d)$ for both, average case better for depth-first $\hookrightarrow$ If you know that there exist many solutions, that the average solution length is rather short and if the branching factor is rather high, use depth-first search, if you are not interested in the optimal but just in some admissible solution.
- Prolog is based on a depth-first search-strategy.
- Typical planning algorithms are depth-first.

Decidability and Complexity

- Decidability:
  - Can we decide for a set of problems *D* (e.g., all classical planning problems)
    - whether a plan exists (PLAN-EXISTENCE(D))
    - whether a solution contains no more than *k* steps (PLAN-LENGTH(D))
- Complexity:
  - How much time or space does it need to decide PLAN-EXISTENCE(D) and PLAN-LENGTH(D)

Motivation

- Recall that in classical planning, even simple problems can have *huge search spaces*
  - Example:
    - ⇒ DWR with five locations, three piles, three robots, 100 containers
    - ⇒ 10^{277} states
    - ⇒ About 10^{190} times as many states as there are particles in universe



- **How difficult is it to solve classical planning problems?**
- The answer depends on which representation scheme we use
  - Classical, set-theoretic, state-variable

Number of States for DWR

An example calculation (our estimate is a bit lower than that of Nau)

5 locations, 3 piles, 3 robots, 100 containers	
occupied(l)	$2^5 = 32$
at(r,l)	$5 \times 4 \times 3 = 60$
loaded(r,c), unloaded(r)	each robot can hold none or one of the containers "n over k" $\frac{101!}{98! \times 3!} = 166.650$
holding(k,c), empty(k)	$\frac{101!}{98! \times 5!} = 8332$
in(c,p),on(c,p), top(c,p)	towers of containers (permutation with piles) $(100 + 14)!$
$\sum$	10^{205}

Decidability & Complexity Analysis

- Complexity analyses are done on *decision problems* or *language-recognition problems* (yes-or-no answers)
- A language is a set $L \subseteq A^*$ of strings over some alphabet A
- A *recognition procedure* for L is a (*possibly non-deterministic*) Turing machine R, so that for all input strings $x \in A^*$:
  - $R(x)$ terminates (*along some computation path*) **and** returns "yes" iff $x \in L$
  - *Notice*: If $x \notin L$, then $R(x)$ may return "no" **or** fail to terminate
- If such a recognition procedure R exists, L is called *Turing-recognizable* (or *semi-decidable* or *recursively enumerable*)
- If R *terminates for all input strings*, then L is called *decidable*
- If L is (semi-)decidable we can look at the *computational complexity* (time and memory consumption of R)

Formal Languages

- A formal language is a set of strings over some alphabet. Given an alphabet A, language L is a subset of all possible words which can be generated form the alphabet. The set of all possible words is equivalent to the elements of the power set over A, that is, 2^{A^*}, $L \subseteq A^*$
- The empty word (contained in A^*) is often denoted with ϵ.
- The empty language is denoted as $\emptyset$.
- Example for $A = \{a, b\}$
  $A^* = \{\epsilon, a, b, aa, ab, ba, bb, aaa, aab, \ldots\}$ a language $L_1 \subseteq A^*$ is for example $L_1 = \{a^i b^i | i \in N_0\}$

Planning as a Language-Recognition Problem

- Translate planning into a language-recognition problem
- Fix a *class of planning problems D*
  - ⇒ Representations: classical, set-theoretic and state-variable
  - ⇒ Domains: Various restrictions and extensions on the language and operators
- Examine the language-recognition problem's decidability and complexity for *D*:

  PLAN-EXISTENCE(D)=
  $\{P|P$ is statement of a planning problem in D that has a solution$\}$
  PLAN-LENGTH(D) =
  $\{(P,k)|P$ is the statement of a planning problem that has a solution of length $\leq k\}$

Basic Decidability Results

- Proposition: For classical, set-theoretic, and state-variable planning PLAN-EXISTENCE(D) is decidable
- Proof idea: If the number of states is finite, we can perform brute-force search to see whether a solution exits
- Proposition: For classical and state-variable planning PLAN-LENGTH(D) is decidable, even if function symbols are allowed
- Proof idea: Do Lifted-backward-search which exits with failure if it reaches a plan of length k which is not a solution. This is a sound procedure which will always terminate. The procedure is also complete: Some execution trace will terminate in $|\pi|$ iterations (proof by induction over the length of π)

Basic Decidability Results

Show that Lifted-backward search is sound:

- Let $P = (O, s_0, g)$ be the statement of a classical planning problem
- let k be a non-negative integer
- Modify procedure such that it exits with failure when a plan of length k is reached which is *not* a solution
- This procedure is **sound**!

Basic Decidability Results

Show that Lifted-backward search is complete:

- Let π be any solution for P of length k or less
- If $|\pi| = 0$, then it is empty and the procedure terminates immediately
- Otherwise, let a_1 be the last action in π
- Then a_1 is *relevant* for g, so it must be a substitution instance of some operator o_i chosen in one of the procedures non-deterministic traces
- $\gamma^{-1}(g, a_1)$ is a substitution instance of $\gamma^{-1}(g, o_1)$
- If $|\pi| = 1$ then s_0 satisfies $\gamma^{-1}(g, a_1)$ and also $\gamma^{-1}(g, o_1)$ and the procedure terminates
- Otherwise, let a_2 be the second-last action in π
- Then a_2 is *relevant* for $\gamma^{-1}(g, a_2)$ ...
- We can show that some execution trace will terminate in $|\pi|$ iterations.

Semi-Decidability (Undecidability) Result

- Proposition: If function symbols are allowed, PLAN-EXISTENCE(D) is semi-decidable **but not decidable.**
- Semi-decidability: It is possible to write a procedure that always terminates with "yes" if $\mathcal{P}$ is solvable and that never returns "yes" if $\mathcal{P}$ is unsolvable. However, if $\mathcal{P}$ is unsolvable, there is no guarantee that the procedure will terminate.

Proof idea: Provability in 1st-order Horn logic can be reduced to classical planning with only positive effects and preconditions:

- A *Horn clause* e:- $p_1, \ldots, p_n$ corresponds to an operator o with $\mathit{precond}(o) = \{p_1, \ldots, p_1\}$ and $\mathit{effect}(o) = \{e\}$
- A goal *g is reachable* iff *g is derivable* in Horn logic from the clauses
- *Derivability in Horn logic undecidable* ([Sebelik & Stepanek 1982]: a single constant and single unary function suffices)

Decidability of Planning

Allow function symbols?	Decidability of *PLAN-EXISTENCE*	Decidability of *PLAN-LENGTH*
no[1]	decidable	decidable
yes	semidecidable[2]	decidable

[1] This is ordinary classical planning.
[2] True even if we make several restrictions.

Language Recognition Problems

- Complexity analyses are done on decision problems or language-recognition problems
- A language is a set L of strings over some alphabet A
- Recognition procedure:
  - $R(x)$ returns “yes“ iff x is in L
  - If x is not in L, the $R(x)$ may return “no“ ore may fail to terminate
- Translate classical planning in a language-recognition problem
- Examine the language-recognition problem’s complexity

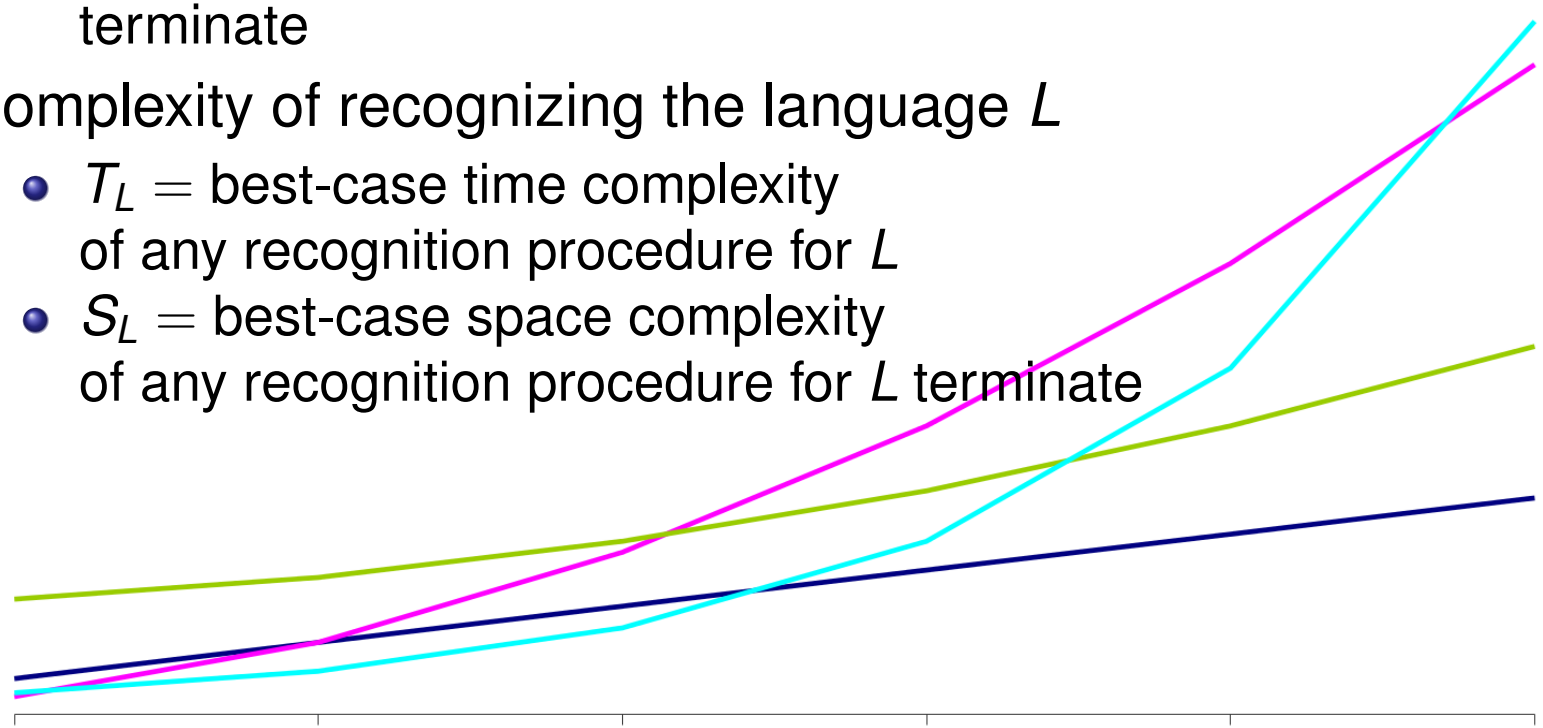
Complexity of Language-Recognition Problems

- Suppose R is a recognition algorithm for a language L
- Complexity of *algorithm R*
  - $T_R(n) =$ *R*'s worst-case time complexity on strings in *L* of length *n*
  - $S_R(n) =$ *R*'s worst-case space complexity on strings in *L* of length *n*

    terminate
- Complexity of recognizing the language *L*
  - $T_L =$ best-case time complexity

    of any recognition procedure for *L*
  - $S_L =$ best-case space complexity

    of any recognition procedure for *L* terminate



O-Calculus

- Computational complexity of procedures are typically analyzed with the O-calculus
- A function $f(n)$ is in the set $O(g(n))$ if there are numbers c and n_0 such that

$$\forall n > n_0, n_0 \leq f(n) \leq c \times g(n)$$

- logarithmically bounded: $f(n) \in O(log\ n)$
- polynomially bounded: $f(n) \in O(n^c)$
- exponentially bounded: $f(n) \in O(c^n)$

Complexity Classes

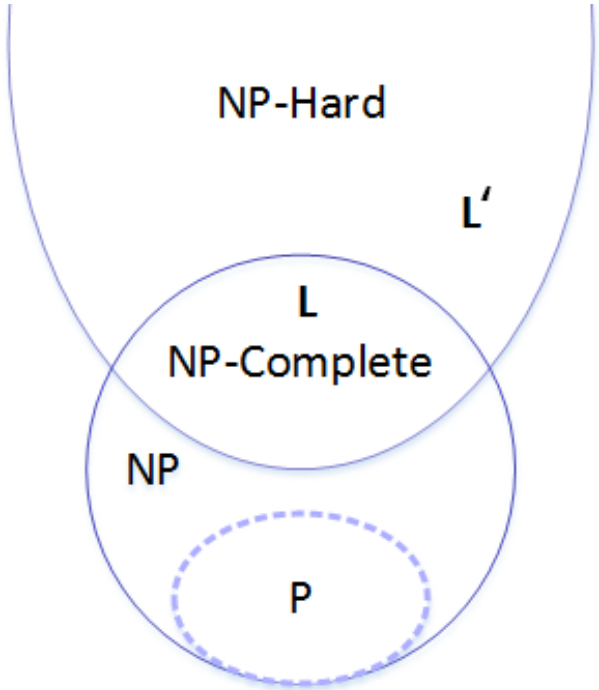
- *Complexity classes:*

	NLOGSPACE	(non-deterministic procedure, logarithmic space)
$\subseteq$	P	(deterministic procedure, polynomial time)
$\subseteq$	NP	(non-deterministic procedure, polynomial time)
$\subseteq$	PSPACE	(deterministic procedure, polynomial space)
$\subseteq$	EXPTIME	(deterministic procedure, exponential time)
$\subseteq$	NEXPTIME	(non-deterministic procedure, exponential time)
$\subseteq$	EXPSPACE	(deterministic procedure, exponential space)

- Let *C* be a complexity class and *L* a language
  - *L* is *C-hard* if for every language *L'* $\in$ C, *L'* can be *reduced* to *L* in a *polynomial amount of time*
    - $\Rightarrow$ NP-hard, PSPACE-hard, etc.
  - *L* is *C-complete* if *L* is C-hard and $L \in C$
    - $\Rightarrow$ NP-complete, PSPACE-complete, etc.

Hardness and Completeness

- **C-complete**:
  L is known to be in C, is a "typical representative"
  (e.g., boolean satisfiability problem, travelling salesman problem)
- **C-hard**:
  every L' can be transformed into $L \in C$ in polynomial time;
  that is: L' is in C or a higher complexity class



Classical Planning is EXPSPACE-Complete

- **Unrestricted classical planning is EXPSPACE-complete.**
- Proof that PLANEXISTENCE is in EXPSPACE:
  Number of ground instances of predicates is *exponential* in terms of input length. Hence, size of a state is at most exponential. Starting with an initial state, we can non-deterministically choose an operator and apply it until we reach the goal. This is NEXPSPACE which is equal to EXPSPACE.
- This holds, if operators are part of the input, that is, not defined in advance (to enable the use of a domain-specific algorithm).

Proof see chap 3.4 in Ghallab, Malik, Dana Nau, and Paolo Traverso. Automated planning: theory & practice. Elsevier, 2004.

D = Classical Planning (CP) without Negative Effects

If there are *no negative effects*, then in searching for a solution plan ...

- ... whenever a closed atom (ground predicate instance) enters the state, it remains part of the state for throughout the plan
- ... no operator needs to be instantiated more than once

Since the *number of* possible *operator instances* is at most **exponential**

- ... we can stop the (depth-first) search after at most **exponentially many steps**

PLANEXISTENCE(D) is in *NEXPTIME*

D = CP without Negative Effects or Negative Preconditions

If, there are neither negative effects *nor negative preconditions*, then

- *operators do not conflict* with each other
- once an operator is applicable, it remains applicable for throughout the plan

Since the *order* in which the operators are applied *does not matter*

- in forward search, simply apply any operator instance that is applicable in the current state and consistent with goal
- we **do not need to backtrack**

PLANEXISTENCE(D) is in *EXPTIME*

D = CP with at Most 1 Precondition or Operators Fixed in Advance

If each operator has **at most one precondition**, then

- ... in backward search, the number of literals in the goal never increases
- ... size of the *current goal* remains *polynomial*

If the operators are **fixed in advance** (arity of operators and predicates), then

- ... there are at most *polynomially many ground atoms*
- ... the size of a state or sub-goal is at most polynomial

PLANEXISTENCE(D) is in *NPSPACE = PSPACE*

- PLAN-LENGTH is never worse than NEXPTIME-complete
  - length may be double-exponential, but we can cut off every search path at depth $k = O(2^n)$

Kind of representation	How the operators are given	Allow negative effects?	Allow negative preconditions?	Complexity of *PLAN-EXISTENCE*	Complexity of *PLAN-LENGTH*
classical rep.	in the input	yes	yes/no	EXPSPACE-complete	NEXPTIME-complete
		no	yes	*NEXPTIME-complete*	*NEXPTIME-complete*
			no	*EXPTIME-complete*	*NEXPTIME-complete*
			no$^\alpha$	*PSPACE-complete*	*PSPACE-complete*
	in advance	yes	yes/no	PSPACE$^\gamma$	PSPACE$^\gamma$
		no	yes	NP$^\gamma$	NP$^\gamma$
			no	P	NP$^\gamma$
			no$^\alpha$	*NLOGSPACE*	NP

Here PLAN-LENGTH is harder than PLAN-EXISTENCE

- In this case, we can write domain-specific algorithms
  - e.g., DWR and Blocks World:

    PLAN-EXISTENCE is in P and PLAN-LENGTH is NP-complete

Kind of representation	How the operators are given	Allow negative effects?	Allow negative preconditions?	Complexity of *PLAN-EXISTENCE*	Complexity of *PLAN-LENGTH*
classical rep.	in the input	yes	yes/no	EXPSPACE-complete	NEXPTIME-complete
		no	yes	*NEXPTIME-complete*	*NEXPTIME-complete*
			no	*EXPTIME-complete*	*NEXPTIME-complete*
			no$^{\alpha}$	*PSPACE-complete*	*PSPACE-complete*
	in advance	yes	yes/no	PSPACE$^{\gamma}$	PSPACE$^{\gamma}$
		no	yes	NP$^{\gamma}$	NP$^{\gamma}$
			no	P	NP$^{\gamma}$
			no$^{\alpha}$	*NLOGSPACE*	NP

no operator has $>$ 1 preconditions

PSPACE-complete or NP-complete for some sets of operators

Remarks

- Often, plan length is harder than plan existence, but it is easier for classical planning (NEXPTIME-complete):
  we can cut of any search path at depth n
- For Tower of Hanoi, the length of the shortest plan can be found in low-order polynomial time, but producing a plan of that length requires exponential time and space

Summary

- Semantics of classical planning is based on interpreting problem representations in a state-space model
- Planning is a calculus, i.e. a syntactic way to compute sound and complete solutions
- If *classical planning* is extended to allow *function symbols*
  - Then we can *encode arbitrary computations* as planning problems
    - $\Rightarrow$ Plan existence is *only* semi-decidable
    - $\Rightarrow$ *Only* plan length is decidable
- Ordinary classical planning is quite complex
    - $\Rightarrow$ Plan existence is *EXPSPACE-complete*
    - $\Rightarrow$ Plan length is *NEXPTIME-complete*
  - But those are *worst case* results
    - $\Rightarrow$ If we can write *domain-specific algorithms*, most well-known planning problems are *much easier*